



Innovation, Science and
Economic Development Canada

Innovation, Sciences et
Développement économique Canada

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Spectrum Management and Telecommunications

Consultation on a Streamlined Framework for Auctioning Residual Spectrum Licences

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Canada 

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1. Intent

1. Through the release of this document, Innovation, Science and Economic Development Canada (ISED), on behalf of the Minister of Innovation, Science and Industry (the Minister), is initiating a consultation on a simplified framework for auctioning residual spectrum licences (referred to as the streamlined framework). This streamlined framework will complement the [Framework for Spectrum Auctions in Canada](#) (FSAC) by establishing an expedited auction process for making unassigned or returned spectrum licences (residual licences) available to interested parties in a timely manner. ISED is seeking comments on implementing a streamlined framework and its parameters, including the auction format and associated rules proposed for future auctions of residual licences.

2. The amount of time required to complete an auction of residual licences can vary, depending on available licences and the amount of time needed to complete a consultation. In some cases, the time to prepare for an auction of residual licences can take as long as the preparation time for the initial auction. ISED is of the view that this process delays getting spectrum into the hands of operators who could put that spectrum to use, while also requiring them to invest their effort and resources into completing a consultation process.

3. For residual licences, the characteristics and conditions of licence would have already been included in consultations on the policy, technical or licensing frameworks during the initial licensing process for those licences. Furthermore, outlining a consistent and predictable format for future auctions of residual licences would make it easier for bidders to participate in the auction. As a result, this streamlined framework seeks to reduce the number of consultations required, simplify any consultations that are required, and promote timely access to the spectrum.

2. Legislative mandate

4. The Minister, through the [Department of Industry Act](#), the [Radiocommunication Act](#) and the [Radiocommunication Regulations](#), with due regard to the objectives of the [Telecommunications Act](#), is responsible for spectrum management in Canada. As such, the Minister is responsible for developing national policies for spectrum utilization and ensuring effective management of the radio frequency spectrum resource.

3. Regulatory background

5. The FSAC describes the general approaches that ISED uses to auction spectrum licences. As a complement to the FSAC, the streamlined framework arising from this consultation will align with the policy objectives of the [Telecommunications Act](#) and the [Spectrum Policy Framework for Canada](#) (SPFC) to maximize the economic and social benefits that Canadians derive from the use of the radio frequency spectrum resource.

4. Context

6. Since 1999, ISED has used auctions to award spectrum licences when the demand for licences was expected to exceed supply. Prior to each process, ISED consults on a variety of factors, including conditions of licence, opening bids, competitive measures, as well as the auction format and the proposed rules. The objective is to provide an opportunity for interested parties to comment on all aspects of the policy related to a specific auction, the auction design, and the licensing procedures and applicable rules. The input received allows ISED to consider the views and perspectives of stakeholders in its decision-making process.

7. In some cases, licences offered in a competitive process remain unassigned at the conclusion of the process. In other cases, previously assigned licences are not renewed or are returned to ISED by a licensee prior to the end of the licence term. To date, ISED has generally made these licences available through a subsequent auction process. These subsequent auctions typically include a full consultation, where stakeholder input is sought on all matters related to the new residual licence auction process, including whether any of the licence characteristics should change from the respective licensing framework that first offered the licences.

8. ISED is of the view that using a traditional consultation process to auction residual licences is overly burdensome for interested stakeholders and could delay timely access to the spectrum. Auctions for residual licences do not require changes to the band plan, and typically do not change the characteristics or conditions of licence from the initial licensing framework. As such, ISED is proposing a streamlined framework for auctioning residual spectrum licences, which minimizes the elements requiring consultation and expedites access to the licences by eliminating the traditional consultation approach and establishing a predictable and consistent format for future auctions of residual licences.

9. Depending on the auction format and rules, some licences that received bids can remain unassigned at the end of the auction process (e.g. as a result of the winner determination process in a combinatorial clock auction). As set out below, ISED is proposing that these residual licences would typically be offered as described in their respective initial licensing framework(s), without changes to their characteristics or relevant conditions. In such cases, ISED would publish a notice of upcoming auction of residual licences, including the information necessary for potential participants. This would limit the need for consultation to situations where ISED deems it appropriate to seek feedback on important changes to the characteristics of the licences or auction rules. Some examples of changes that would not require consultation are set out below. In such cases, ISED would launch a shorter consultation that targets the specific changes being considered before publishing the notice of upcoming auction of residual licences.

10. Recognizing that stakeholders should be given an opportunity to provide feedback on this expedited process, this consultation seeks comments on the streamlined framework and the proposed approach to streamlining future auctions of residual licences, including general timelines, auction format and rules.

5. Approach to streamlining auctions of residual spectrum licences

11. This section provides details on the residual licences that would be included in an auction and outlines the proposed approach of this streamlined process.

5.1 Applicable licences

12. The proposed streamlined framework for auctioning residual spectrum licences would be restricted to auctioning residual spectrum licences, defined as either:

- spectrum licences that remain unassigned at the conclusion of an auction process
- spectrum licences that have not been renewed, were previously assigned but have been cancelled by ISED, or have been returned to ISED by the licensee prior to the end of the licence term

5.2 Licence characteristics

13. Typically, the band plan and available licences are consulted on and determined as part of the respective initial licensing framework(s) that first offered the licences. This includes available blocks, block sizes, and tier sizes. In order to remain consistent with the licences awarded through the initial licensing process, block sizes would typically remain the same as established in the initial licensing framework. However, based on observed demand, ISED may deem it necessary to modify other licence characteristics such as tier size or opening bid prices.

5.2.1 Tier sizes

14. The range of tier sizes that could be used in a spectrum auction are outlined in the [Service areas for competitive licensing](#) interactive map and table. The tier size for a specific set of licences is typically determined as part of consultations for the relevant band. For some providers, smaller tier sizes could be more appealing as they might reflect their particular business case more closely, while becoming available at a relatively lower cost.

15. Depending on the level of demand observed during the previous licensing process (very low or nil), ISED may deem it appropriate to modify the tier size for residual licences. For example, after observing low demand for the licences at the Tier 2 level in service area 2-014 Yukon, Northwest Territories & Nunavut during the 700 MHz auction in 2014, ISED offered those licences at the Tier 4 level (4-170 Yukon, 4-171 Nunavut and 4-172 Northwest Territories) in two subsequent auctions of residual licences.

16. For an auction of residual licences, ISED may consider modifying tier sizes to a smaller tier size in order to reduce the area of coverage, which would effectively reduce the relative opening bid price of the licence. Given that the tier sizes would only be modified for licences where there is already evidence of low (or nil) demand, ISED proposes to not consult on the question of tier size for residual licences. Updates to opening bid prices that may result from changes to tier sizes are discussed in section 5.2.2.

5.2.2 Opening bid prices

17. Opening bid prices represent the prices for spectrum licences at the start of the auction, and the minimum amount that will be accepted for each licence (reserve price). They are set high enough to ensure that Canadians receive a fair return for the spectrum, but at a level that should not discourage auction participation from operators who would want to put that spectrum to use.

18. Opening bid prices for residual spectrum licences have typically been set at the level established in the licensing process that initially offered the respective licences. However, in circumstances where ISED has observed very low (or nil) demand for the licences, it may be appropriate to adjust the opening bid prices.

19. For an auction of residual licences, ISED would only consider maintaining or reducing, not increasing, opening bid prices depending on observed demand. This could be a proportional reduction in prices due to changes in the characteristics of the licences (for example, if licences are offered at a smaller tier size or if conditions of licence are changed) or simply a reduction in the minimum price to increase demand for the licences. Since opening bid prices from the relevant initial auction processes have already been consulted on and represent the maximum possible opening bid prices for an auction of residual licences, ISED proposes to not consult on opening bid prices prior to auctioning residual licences.

Q1

ISED is seeking comments on its proposals to not consult on the following as part of developing the streamlined framework for auctioning residual spectrum licences:

- a. potential changes to tier sizes when auctioning residual licences
- b. opening bid prices that are equal to or lower than the opening bid prices from the initial licensing framework(s) for those licences

5.3 Pro-competitive measures

20. As noted in section 4 of the [Framework for Spectrum Auctions in Canada](#), promoting a competitive post-auction marketplace is an important part of maximizing the social and economic benefits derived from the use of the radio frequency spectrum.

21. The FSAC outlines two guiding principles for choosing when and how to impose pro-competitive measures, either by specifying who can participate in the wireless market and/or through spectrum aggregation limits. In past auctions, ISED has notably used set-asides to ensure that a minimum amount of spectrum is reserved for specific types of entities (e.g. AWS-3 in 2015, 600 MHz in 2019); and spectrum aggregation limits to cap the amount of spectrum that each licensee is allowed to obtain (e.g. 700 MHz in 2014, 2500 MHz in 2015).

22. ISED is of the view that competitive measures used in initial licensing processes would generally remain relevant to address issues of market power and to promote a competitive post-auction marketplace during subsequent allocation processes. As such, residual licences will typically be subject to the competitive measures established in the licensing framework that initially offered those licences. However, depending on the demand observed during the initial

allocation process or the time that lapses between the initial allocation process and the auction of residual licences, ISED may deem it necessary to change the applicable competitive measures for some of the licences being offered.

23. ISED proposes only to consult on competitive measures for residual licences when it is considering competitive measures that differ significantly from the measures used in the initial licensing process for those licences. In such cases, ISED would launch a short consultation to provide stakeholders with an opportunity to comment on the proposed competitive measures.

Q2

ISED is seeking comments on its proposal to only consult on competitive measures for an auction of residual spectrum licences when it is considering competitive measures that differ significantly from the measures used in the initial licensing process for those licences.

5.4 Conditions of licence

24. ISED typically consults on the conditions of licence as part of each licensing process. This includes conditions such as licence term, transferability, divisibility and subordinate licensing, deployment conditions, eligibility, treatment of existing spectrum users, technical considerations, lawful interception, research and development, and other conditions.

25. In previous auctions of residual licences, ISED typically maintained the same conditions of licence that were established in the initial licensing framework for those licences. In order to reduce the number of elements requiring consultation, ISED proposes to not consult when the conditions of licence for residual licences are unchanged from the conditions established in the licensing framework that initially offered those licences.

26. In some other cases, a condition of licence may have changed following a wider consultation on a condition that applies across a number of bands, such as provisions on tower siting, tower sharing, or roaming. Under this streamlined framework, ISED is also proposing not to consult in cases where a condition of licence has been amended as a result of a prior consultation.

27. It should be noted that licences are always subject to the relevant provisions in the [Radiocommunication Act](#) and the [Radiocommunication Regulations](#). As such, the Minister continues to have the power to amend the terms and conditions of spectrum licences pursuant to paragraph 5(1)(b) of the *Radiocommunication Act*. Depending on the demand for the licences observed during the initial licensing process, ISED may deem it necessary to modify some of the conditions of licence, such as, but not limited to, licence term and/or deployment requirements.

5.4.1 Licence term

28. Longer licence terms have historically been favoured in order to attract the financial investment required to deploy next generation wireless services. However, there may be cases where licensees prefer to receive residual licences with a term that aligns with licences acquired during the initial licensing process for those licences.

29. In order to simplify the planning process and to prevent future complications, where applicable, ISED proposes to amend the licence term for licences obtained in an auction of residual licences so the expiration date aligns with the expiration date of the first licences issued following the auction for that band. As a result of this alignment, licences in a specific band would expire on approximately the same date whether the licence was obtained in the initial licensing process for that band or a subsequent auction of residual licences. For example, if the first licence issued after the initial auction process for a given band expires on December 31, 2030, then all residual licences, regardless of when they were issued, would also expire on that date.

5.4.2 Deployment requirements

30. ISED recognizes that observed demand for the licences (very low or nil), or modifications to the characteristics or conditions of licence (such as tier size or licence term) may make it appropriate to consider amendments to deployment requirements. As such, ISED proposes to not consult on establishing amended deployment requirements for residual licences in order to respond to observed demand or to complement a change to the characteristics or the conditions of the licence.

Q3

ISED is seeking comments on its proposal to not consult on the following as part of the process for auctioning residual spectrum licences:

- a. the applicable conditions of licence when they are the same as the conditions established in the initial licensing process offering those licences or have been amended as a result of another consultation
- b. the term of licence for residual licences when the term is modified to align their expiration date to the same approximate expiration date of licences awarded through the initial licensing process for that same band
- c. amended deployment requirements when they are modified to respond to observed demand or to correspond to changes in the characteristics or conditions of the licence

5.5 Auction design

As indicated in the FSAC, auctions are an efficient market-based means of assigning spectrum licences, through a fair and transparent process, to those who value them most. ISED's objective is to select an auction design that is optimal for the spectrum being offered and the circumstances that exist at the time. In this context, ISED proposes the following design for an auction of residual licences.

5.5.1 Sealed-bid auction

31. For a streamlined process, the auction format should be fair, easy to understand, and not too burdensome for bidders to participate in. The auction format should result in an efficient assignment of spectrum. In selecting a format and rules, ISED considers the characteristics of the available licences, such as block size, tier size and potential complementarities across blocks.

32. For past auctions of residual spectrum licences, ISED used the sealed-bid auction format. In this format, bidders simultaneously submit sealed bids on the licence(s) they are interested in, so that no bidder knows the bids of any other participant. To mitigate the possibility of a tie, bidders are encouraged not to bid in round figures. In the event of a tie, tied bidders will be requested to enter a second sealed bid in an attempt to break the tie. Should there be subsequent ties, bidders will be requested to continue submitting sealed bids until there is no tie.

33. Although other formats provide stakeholders with the benefit of price discovery through multiple rounds, those formats are more complex and time consuming for stakeholders. Given the limited number of licences that would be available in an auction of residual licences, ISED is of the view that a sealed-bid format is well-suited to making the licences available in a timely manner. ISED therefore proposes that the streamlined framework use a sealed-bid format for future auctions of residual licences.

5.5.2 Combinatorial (package) bidding

34. In a single round sealed-bid auction, where bidders bid for specific licences individually, the highest bidder wins the licence. However, when a larger number of licences is made available across many service areas, there is a risk that a bidder may win some, but not all of the licences required for its business plan (called exposure risk). Allowing for combinatorial (package) bidding could mitigate exposure risk by allowing bidders to submit bids that express their values for the complementarities or interdependencies that might exist between licences in different geographic areas or across different frequencies.

35. With package bidding, bidders would be given the opportunity to submit all-or-nothing bids for collections of licences where each bid could be won only in its entirety or not at all. This would prevent winning bidders from acquiring only a subset of licences from packages it had bid on in the auction.

36. ISED proposes to retain the option to include package bidding in an auction for residual licences when deemed appropriate, and to not consult before doing so.

5.5.3 Second-price rule

37. In recent auctions (e.g. 700 MHz, 2500 MHz, 600 MHz, and both recent auctions of residual licences), ISED has used a second-price rule to determine the prices that winning bidders would be required to pay. This results in each winning bidder paying a sufficient amount to ensure that no other bidder (or group of bidders) was willing to pay more for the licences won.

38. **Auction without package bidding:** When bidding is only for individual licences, licences will be awarded to the highest bidder for each licence, and the Vickrey price for winning bidders will be the second highest bid on each licence, or the opening bid price in the absence of other bids.

39. **Auction with package bidding:** When bidding for packages of licences is allowed, licences are awarded to the bidders that collectively submitted the combination of bids that yields the highest value, limited to one winning bid per bidder. In this case, prices could depend on other package bids, bids for individual licences, and/or the opening bid prices. Further details on how prices are calculated when package bidding is included are provided in annex A.

40. An alternative to the second-price rule is the first-price rule (or “pay-as-bid”), where bidders pay the full amount of their bid for the licences they win. However, this introduces the risk of incentivizing bidders to bid less than their true value, potentially leading to inefficient outcomes. By ensuring that winning bidders will have to pay the minimum amount necessary to win their desired licence(s), a second-price rule should motivate truthful bidding during the auction. As such, ISED proposes that future auctions of residual licences use a second-price rule to calculate prices.

Q4

ISED is seeking comments on its proposal to use a sealed-bid auction format with a second-price rule to auction residual licences, and to retain the option to allow combinatorial (package) bidding when ISED deems it appropriate.

5.6 Unallocated licences

41. Licences that remain unassigned following an auction of residual licences will be retained by ISED and consideration will be given to making them available through an alternative process at a later date. The timing and form of such a process will depend on the demand for the licences.

5.7 Auction process and bidder participation

42. In general, ISED consults on the proposed process for submitting an application to participate in the auction, as well as the general requirements and rules that would apply prior to, during and post-auction as part of the initial licensing process. In terms of bidder participation, this includes rules regarding affiliated and associated entities, the prohibition of collusion and other communications, and auction integrity and transparency. It also includes, but is not limited to, information on the auction process, such as how to apply to participate, what to include in a submission, pre-auction financial deposits, bidder qualification, how to submit bids, the determination of provisional licence winners, initial and final payments, and forfeiture penalties.

43. Each auction of residual licences would typically reflect the auction process and rules for bidder participation established as part of the respective initial licensing framework(s) for the available licences. In such cases, ISED proposes to not consult on the elements of auction process and bidder participation prior to the launch of an auction of residual licences as they have already been consulted on.

44. ISED proposes to publish a notice of upcoming auction of residual licences that details the residual licences available (blocks and tier sizes), the competitive measures, the opening bid prices, the conditions of licence, information on the auction process including a Table of Key Dates, and the rules for participating in the auction.

5.7.1 Timing (Table of key dates)

45. Given the simplified process, the streamlined framework would allow ISED to reduce the amount of time required to implement an auction of residual spectrum licences. However, ISED acknowledges that potential auction participants may require a certain amount of time to evaluate the licences being offered, to prepare their business plans and valuations, and to seek funding to participate in the auction. As such, ISED is seeking stakeholder feedback on the amount of time that potential bidders would require to prepare for an auction of residual licences. A proposed table of key dates (table 1), with estimated timelines, is shown for stakeholder reference.

Table 1: Proposed table of key dates

Event	Estimated timeline (Business days from previous event)
Publication of notice of upcoming auction of residual licences	Publication date
Deadline for submitting clarification questions	+ 15 days
Publication of clarification questions and responses	+ 15 days
Deadline for submitting applications to participate and 100% pre-auction financial deposit	+ 25 days
Publication of list of applicants, beneficial ownership and associated entities information	+ 5 days
Publication of list of qualified bidders and Auction documentation sent to qualified bidders	+ 10 days
Sealed bid deadline	+ 25 days
Announcement and publication of provisional winners of licences	+ 5 days
Initial payment (20% of total final payment)	+ 10 days
Final payment (remaining 80% of total final payment)	+ 20 days
Total number of business days from publication	130 days

46. The above timeline reflects an estimate of the amount of time required to implement an auction of residual licences from the date of publication when no consultation is required. For each auction of residual licences, the exact dates would be established in the table of key dates published in the notice of upcoming auction of residual licences. However, if ISED opts to consult prior to an auction of residual licences, additional time would be required to complete the consultation. In such cases, the consultation would include a new proposed table of key dates, with an opportunity for stakeholders to provide comments. Note that in either case, ISED may opt to update the Table of Key Dates without consultation in order to accommodate any unexpected circumstances.

Q5

ISED is seeking comments on the amount of time potential bidders require to prepare for an auction of residual licences and the timelines outlined in the proposed Table of Key Dates.

Q6

ISED is seeking comments on its proposal to not consult on elements of the auction process and the rules for bidder participation prior to an auction for residual licences.

6. Submitting comments

47. All submissions should cite the *Canada Gazette*, Part I, the publication date, the title and the notice reference number (SLPB-001-21). Parties should submit their comments no later than **May 17, 2021** to ensure consideration. Soon after the close of the comment period, all comments received will be posted on ISED's [Spectrum Management and Telecommunications](https://www.spectrum.gc.ca/en/telecom/spectrum-management) website.

48. Electronic submissions (Microsoft Word or Adobe PDF) should be sent via email to ic.spectrumauctions-encheresduspectre.ic@canada.ca.

49. Paper submissions should be mailed to the following address:

Innovation, Science and Economic Development Canada
Senior Director, Spectrum Licensing and Policy Branch
235 Queen Street (6th Floor, East Tower)
Ottawa ON K1A 0H5

50. In addition, respondents are asked to specify question numbers for ease of referencing and to provide supporting rationale for each response. As all comments will be published, respondents are asked to not include confidential or private information in their submissions.

51. Following the initial comment period, ISED may, at its discretion, request additional information if needed to clarify significant positions or new proposals.

7. Obtaining copies

52. All spectrum-related documents referred to in this paper are available on ISED's [Spectrum Management and Telecommunications](#) website.

53. For further information concerning the process outlined in this document or related matters, contact:

Innovation, Science and Economic Development Canada
c/o Senior Director, Spectrum Licensing Policy Branch
235 Queen Street (6th Floor, East Tower)
Ottawa ON K1A 0H5
Telephone: 343-291-1400
TTY: 1-866-694-8389
Email: ic.spectrumauctions-encheresduspectre.ic@canada.ca

Annex A: Proposed second-price rule for combinatorial (package) bidding

The proposed second-price rule would determine the prices to be paid such that the price for a winning bidder will be at least the opening bid price, but no higher than the submitted bid amount. Second prices are often referred to as Vickrey prices and represent the opportunity cost of the bidder winning the licence or package. More specifically, ISED proposes to apply bidder-optimal core pricing using the nearest Vickrey approach.

When licences are made available for package bidding, bids can be made for individual licences and/or a combination of the available licences. Winning bids will be calculated by determining the combination of bids that generates the highest sum.

The Vickrey price(s) could depend on package bids and/or bids for the individual licences as well as the opening bid prices for the licences. In addition, with package bidding, an extra payment beyond Vickrey prices is sometimes required as a result of interaction between overlapping bids. In the event that an extra payment is required, the payment to be made will be adjusted so that it is proportionate to the size of the bidder's package, in terms of value at opening bid prices.

The second or Vickrey price for each winning bidder (known as "Bidder J" in this explanation) is calculated as follows. First, subtract Bidder J's winning bid (value A) from the value of the winning combination. Next, recalculate the winning combination for the hypothetical situation in which all Bidder J's bids are excluded, as if Bidder J had not participated (value B). The Vickrey price for Bidder J is calculated as the value of the winning combination with all Bidder J's bids excluded (value B) minus the sum of the winning bids for all bidders other than Bidder J (value A); that is, value B minus value A. This is the minimum amount that the winning bidder could have bid in order to still have won, given the bids of all other bidders.

The prices to be paid by winning bidders must satisfy the following conditions:

- a. **First condition:** The price for a winning bid must be greater than or equal to the opening bid price(s) for the licence(s) in the bid, but not more than the dollar amount of the winning bid.
- b. **Second condition:** The set of prices must be sufficiently high that there is no alternative bidder, or group of bidders, prepared to pay more than any winning bidder or group of winning bidders. If there is only one set of prices that meets the first and second conditions, this determines the prices to be paid.
- c. **Third condition:** If there is more than one set of prices that fulfils the first and second conditions, the set (or sets) of prices minimizing the sum of prices across winning bidders is (are) selected. If there is only one set of prices satisfying these three conditions, this set determines the prices.

- d. **Fourth condition:** If there is more than one set of prices that satisfies the first three conditions, the set of prices that minimizes the weighted sum of squares of differences between the prices and the Vickrey prices will be selected. The weighting is relative to the price of the bidder's package evaluated at opening bid prices. This approach for selecting among sets of prices that minimize the sum of prices across winning bidders is referred to as the "nearest Vickrey" approach.

These conditions characterize a unique set of prices such that each winning bidder pays no more than the dollar amount of its winning bid and pays at least the opening bid price(s). ISED staff will calculate the set of prices that meet the conditions outlined above.

The following is an example of how prices are calculated, based on [Spectrum Auction Design](#) by P. Cramton.

Suppose five bidders, 1, 2, 3, 4 and 5, are bidding for two licences, A and B. The following bids are submitted ("b" designates the bidder):

- $b_1\{A\} = \$28$
- $b_2\{B\} = \$20$
- $b_3\{AB\} = \$32$
- $b_4\{A\} = \$14$
- $b_5\{B\} = \$12$

The bids of the five bidders are represented in figure A1.

In this example, the highest value combination of bids would assign licence A to Bidder 1 and licence B to Bidder 2, generating \$48 in value. There is no other assignment of licences that yields a higher value.

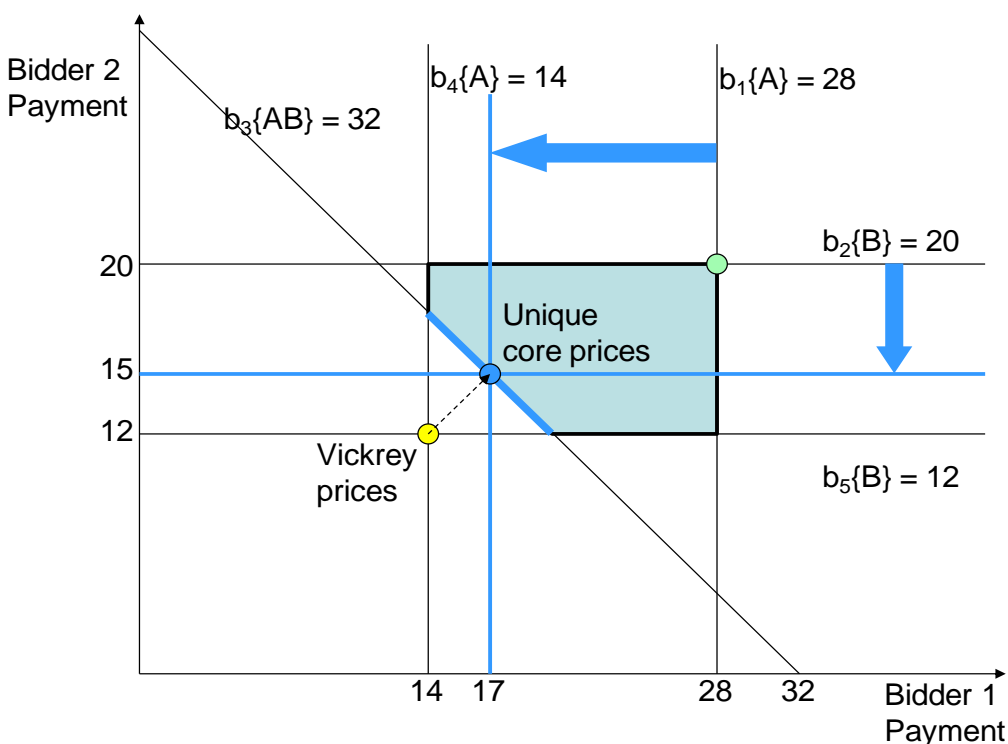
To calculate the Vickrey price for Bidder 1, its winning bid (\$28) is subtracted from the value of the winning combination (\$48), resulting in \$20. Next, the winning combination is recalculated for the hypothetical situation in which Bidder 1's bids are excluded. The best assignment, excluding Bidder 1, assigns licence A to Bidder 4 at \$14 and licence B to Bidder 2 at \$20, resulting in \$34. The Vickrey price for Bidder 1 is the value of the winning combination of packages with all Bidder 1's bids excluded (\$34) less the sum of the winning allocation stage bids for all bidders other than Bidder 1 (\$20); that is, its Vickrey price is \$14 (\$34 – \$20).

Similarly, to calculate the Vickrey price for Bidder 2, its winning bid (\$20) is subtracted from the value of the winning combination (\$48), resulting in \$28. Next, the winning combination is recalculated for the hypothetical situation in which Bidder 2's bids are excluded. The best assignment, excluding Bidder 2, assigns licence A to Bidder 1 and licence B to Bidder 5, resulting in a value of \$40. The Vickrey price for Bidder 2 is the value of the winning combination of packages with all Bidder 2's bids excluded (\$40) less the sum of the winning

allocation stage bids for all bidders other than Bidder 2 (\$28); that is, its Vickrey price is \$12 (\$40 - \$28).

Hence, the Vickrey outcome is for Bidder 1 to pay \$14 for licence A and for Bidder 2 to pay \$12 for licence B. Total revenues with these payments are \$14 + \$12 = \$26. As shown in Figure A1, this means that Bidder 1 can reduce its bid to \$14 before being displaced by Bidder 4. Similarly, Bidder 2 can reduce its bid to \$12 before being displaced by Bidder 5.

Figure A1: Example of calculating prices (in dollars)



However, these payments sum to \$26, which is less than Bidder 3’s bid of \$32 for both licences A and B. Therefore, Bidder 1 and Bidder 2 must split an additional payment of \$6 (\$32 - \$26), to ensure that their combined payment is greater than that of Bidder 3, satisfying the condition that no other bidder or group of bidders was prepared to pay more for the licences in question. That is, Bidder 1 and Bidder 2 must pay, collectively, at least \$32.

If the opening bid prices for licence A and licence B are the same amount, the additional payment of \$6 is split equally between the two bidders. Each bidder is therefore paying an additional \$3 above its Vickrey price, with Bidder 1 paying \$17 (\$14 + \$3) and Bidder 2 paying \$15 (\$12 + \$3), as shown in figure A1.

However, if the opening bid prices for the two licences are different amounts, the two bidders must split the extra payment proportionately to the opening bid amounts (the fourth condition). For example, if the opening bid price for licence A is \$8 and the opening bid price for licence B

is \$4, then the opening bid price of Bidder 1's package is twice as large as that of Bidder 2. Therefore, Bidder 1 would pay twice as much as Bidder 2 of the extra payment, with Bidder 1 paying an additional \$4, for a total payment of \$18 and Bidder 2 paying an additional \$2, for a final payment of \$14.

In the event of a tie during the second price calculation, a pseudorandom generator will be used to decide between tied outcomes.